Eur. Phys. J. B **32**, 309–314 (2003)
DOI: 10.1140/epjb/e2003-00103-x

THE EUROPEAN PHYSICAL JOURNAL B

Location of the minimum of the differential tunneling resistance R(V) in a superconductor-degenerate semiconductor Schottky contact

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Received 11 November 2002 / Received in final form 21 February 2003
Published online 11 April 2003 –

Abstract. Measurements of differential resistance in a superconductor-degenerate semiconductor junction $\mathrm{Nb}-n^{++}\mathrm{GaAs}$ at $T = 1.6$ K show close similarity to those for a conventional superconductor-insulator-normal metal junction, except for the position of the minimum which is located at 3.6 meV. Using a simple model for the charge screening at the Schottky barrier, we give an argument why this minimum is by far displaced with respect to the superconducting gap energy ($\Delta_g = 1.5$ meV for bulk Nb). We argue that a rebuilding of the density of states takes place at the barrier, due to the imperfect metal screening in the degenerate semiconductor. Energy states close to the degenerate semiconductor Fermi energy are depleted at the barrier and are not available for tunneling, up to an energy E_g which adds to the superconducting gap Δ_g.

PACS. 73.30.+y Surface double layers, Schottky barriers, and work functions – 73.40.-c Electronic transport in interface structures – 74.80.Fp Point contacts; SN and SNS junctions

1 Introduction

Superconductor-semiconductor ($S - Sm$) contacts have been intensively investigated both experimentally and theoretically for over 30 years [1]. In the 1980s and 1990s, a remarkable effort was done in order to complete the physical pictures of this type of contact in different structures: simple $S - Sm$ contacts [2,3], superconducting field effect transistor [4], sandwich-type $S - Sm - S$ membrane Josephson junctions [5], $S{-}Sm{-}S$ light sensitive semiconducting barrier junctions [6]. In recent years the topic attracts new interest because of the promising role of hybrid systems in technological advances. In particular, a new class of $S - Sm$ contacts is investigated, namely junctions between a superconductor and a two-dimensional electron gas in semiconductor heterostructures [7].

It is well known that, when a metal (normal or superconducting) is in contact with a semiconductor, a Schottky barrier is formed at the interface and the interface transmittance can be changed over many orders of magnitude by varying the Sm doping [8]. Recently, considerable efforts are being devoted to the study of the superconductor-degenerate semiconductor ($S - dSm$) junction, when the carrier density in the semiconductor is so high, that the latter plays the role of a normal metal with a Fermi level μ in the conduction band [9]. As a matter of fact, the Andreev conduction is observed in such contacts [10].

In normal metal-superconductor ($N - S$) point contact structures, the Blonder, Tinkham and Klapwijk (BTK) model [11] for current transport applies satisfactorily [2]. Within the same model, the minimum of the differential resistance $R(V) = \mathrm{d}V/\mathrm{d}I$ for superconductor- insulator - normal metal (S–I–N) junctions should be located at the voltage $V = \Delta_g/e$, that is at the superconducting energy gap. This is usually well confirmed by experiments [10].

If the same picture holds for the $S - dSm$ junctions, the minimum of $R(V)$ should here be found at $V = \Delta_g/e$, as well. Surprisingly, we have collected quite a few piece of evidence, including data by different experimental groups in various $S - dSm$ contacts, that show in a convincing way that this is not the case. The $R(V)$ dependence shows that the minimum is markedly shifted with respect to the value of the voltage corresponding to Δ_g.

The anomaly was reported in measurements of various contacts: $\mathrm{Sn} - p^{+}\mathrm{GaAs}$ [2], $\mathrm{Nb} - n^{++}\mathrm{Si}$ [12], $\mathrm{Nb} - n^{++}\mathrm{GaAs}$ [3]. The carrier density in the mentioned degenerate Sm contacts ranges between $n = 2{\times}10^{+18}$ cm^{-3} [2] and $n = 10^{+19}$ cm^{-3} [3]. This finding is usually notified without offering any explanation. The authors of reference [2] make the conclusion that the minimum is not a good measure of Δ_g in $S - Sm$ junctions.

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The shift of the minimum in $R(V)$ does not seem to fit with the BTK picture [2] and urges for interpretation.

In this paper we exclude that the observed shift is due to an increase of Δ_g. As explained in Appendix A, the superconducting coherence in the metal side of the interface plays a minor role in the screening, as the typical screening length on the superconducting side is the Debye length of full metal screening, λ_D, which is quite small. Hence, we infer that the superconducting properties do not influence much the Schottky barrier at the interface.

Instead, we attribute the shift of the minimum of $R(V)$ to the peculiar features of the Schottky barrier charge screening on the Sm side of the $S - dSm$ interface.

In Section 2 we present the experimental findings and we outline our interpretation.

In Section 3, we solve a simple one-dimensional barrier model which, in the spirit of the BTK approach, is intended to mimic the depletion layer. We show that the energy dependence of the density of states is modified by the single particle scattering against the barrier. The modification takes the form of a lack of states available for tunneling, up to an energy E_g which can be of the order of Δ_g itself. Indeed, for an opaque barrier, quite a few states originally below μ are moved to higher energies, which produces an enhancement of the conduction at the voltage $V \sim (E_g + \Delta_g)/e$.

In Section 4 we discuss the interpretation of the experimental findings provided by our model.

2 Experimental observation of the differential resistance and its qualitative explanation

In this section we report on measurements of $R(V)$ performed by some of us [3] in $\mathrm{Nb} - n^{++}\mathrm{GaAs}$ contacts.

The contact was done between the Nb film and the n^{++}GaAs layer with the Si impurity concentration $n = 10^{+19}$ cm^{-3}. Figure 1 shows the schematic of the sample. The Si impurity concentration of the n^{+}GaAs layer was about $5\times10^{+17}$ cm^{-3}. The dashed line in Figure 1 shows penetration of the $\mathrm{Au-Ge-Ni}$ alloy into the GaAs material after annealing. The junction resistance was measured using a conventional four terminal lock-in technique. Since the high-frequency field-effect transistor geometry with small contact paths was used, it was possible to bond only one Al wiring both to the Nb electrode and to the $\mathrm{Au - Ge - Ni}$ ohmic contact of the n^{++}GaAs layer. One large Au chip carrier path was connected with the Nb electrode and another one was connected with the $\mathrm{Au{-}Ge{-}Ni}$ ohmic contact of the n^{++}GaAs layer. The two copper terminals of the electronic set-up were attached to these chip carrier Au paths. Further details on the sample fabrication can be found in reference [3].

Figure 2a shows the dependence of the dynamic resistance on the applied voltage of the $\mathrm{Nb}-n^{++}\mathrm{GaAs}$ contact measured at $T = 1.6$ K. On the measured $R(V)$ curve a strong resistance peak occurs near zero bias and a minimum appears around 3.6 meV at a bias current of 30 μA. This is a big shift with respect to an expected minimum to be measured at the superconducting energy gap of the bulk Nb ($\Delta_g = 1.5$ meV). We exclude the possibility to explain the observed shift by the presence of a series resistance in n^{++}GaAs material. This material has a length of 1 μm and is located between $\mathrm{Au-Ge-Ni}$ and Nb contacts (see Fig. 1). A simple estimation gives for the associated series resistance a value of about 0.55 Ω [8], which is too low to justify the observed volage shift at a bias current of

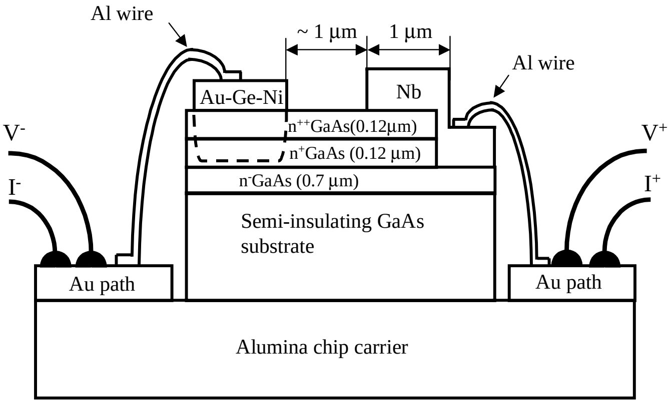

Fig. 1. Schematic of the sample (Ref. [3]). Cross-section is cut along the short dimension (1μm) of the Nb contact. The long dimension of the Nb contact is 60 μm.

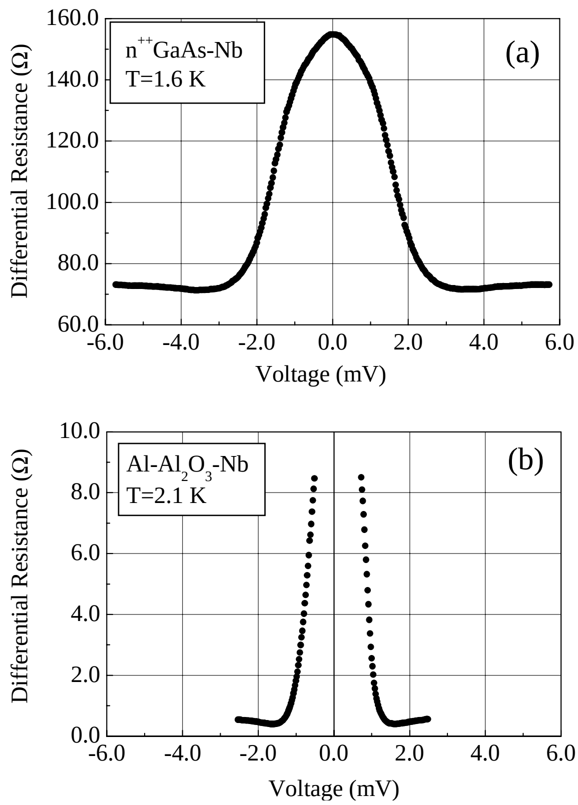

Fig. 2. The differential resistance $R(V)$ in a $\mathrm{Nb}-n^{++}\mathrm{GaAs}$ contact (a). The $R(V)$ in a $\mathrm{Al\text{-}Al_2O_3\text{-}Nb}$ contact fabricated and measured under the same conditions is reported for comparison (b). The latter clearly shows that the minimum is located at the voltage corresponding to the superconducting energy gap.

30 μA. In order to test the reliability of $R(V)$ as measured by our apparatus, the same measurements were performed on $Al - Al_2O_3 - Nb$ junctions at $T = 2.1$ K and the result is shown in Figure 2b. Actually, the $R(V)$ curve of Figure 2b is the typical one to be expected in the case of a conventional S–I–N tunnel junction. The voltage position of the minimum is 1.6 meV and it coincides with the superconducting energy gap of the bulk Nb.

Qualitatively, the shape of the $R(V)$ dependence of the conventional S–I–N junction (Fig. 2b) is similar to the $R(V)$ curve of the $Nb - n^{++}GaAs$ contact. This shows that the n^{++}GaAs layer investigated in [3] can be considered as a normal metal. Indeed, GaAs with a concentration of dopants of $n = 10^{+19}$ cm^{-3} is a strongly degenerate semiconductor in the temperature range from $T = 0$ K up to $T = 300$ K. The Fermi level μ, measured from the bottom of the conductance band E_c, can be simply estimated as if the degenerate GaAs were a Drude metal [9]:

$$n = (8\pi/3)(2m_n^*/h^2)^{3/2}(\mu - E_c)^{3/2}. \qquad (1)$$

For the sample of reference [3], equation (1) gives the value of $\mu - E_c \simeq 253$ meV.

We claim that the observed shift can be explained from the point of view of the charge screening at the Schottky barrier of the $S - dSm$ interface. What makes the difference with respect to a S–I–N junction is the imperfect metal screening in the degenerate semiconductor which renders the energy states close to the Fermi energy not available for tunneling. Because of the presence of the Schottky barrier, there is a depletion of tunneling states close to the interface on the Sm side, up to an energy E_g. Therefore, the experimentally measured minimum in $\mathrm{d}V/\mathrm{d}I$ is shifted with respect to $V = 0$ of the quantity $(E_g + \Delta_g)/e$. Our qualitative model allows us to give a simple numerical estimate for E_g, of the order of the measured value.

The outline of our argument is the following. Let us first consider the $N - Sm$ junction. A Schottky barrier forms with a double layer of charge at the interface. Negative charge is accumulated on the metal side while mobile negative carriers are depleted on the semiconducting side, over a distance $w \approx 8 \div 12{\times}10^{-7}$ cm calculated for our experimental situation of $n = 10^{+19}$ cm^{-3} [8] (see Fig. 3). The fact that the metal side is superconducting has minor effects at equilibrium, because of the full screening of the charge accumulated on the S side. This is shown by us in an elementary way in Appendix A, using a two fluid picture. Screening occurs on distances of the order of the Debye length λ_D as if the metal were normal conducting. This length is much smaller than the coherence length of Nb. Our macroscopic approach could be questionable, if subgap conductance takes place, what we think is not the case here. Proximity effects, in the form of a zero voltage anomaly due to Andreev scattering, are presently studied in very transparent junctions. On the contrary, we consider finite bias differential resistance here, which suggests that the barrier in these samples is quite opaque to tunneling.

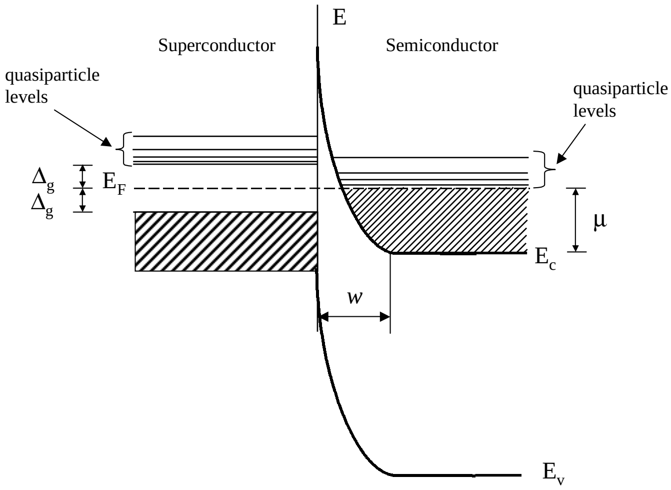

Fig. 3. Sketch of the energy-bands with band bending at the superconductor-n-type semiconductor contact.

We conclude that the superconductor plays the same role in $S - dSm$ contacts as in S–I–N junctions, and we take the semiconductor for responsible of the anomaly in the minimum position, due to imperfect screening. How this happens, is described in the following.

Being the bulk semiconductor heavily doped such to be degenerate, the Fermi energy level is located above the bottom of conduction band and a Fermi sea of mobile carriers is formed. Some of the carriers are involved in the screening of the electric field produced by the Schottky barrier formation, very much like what Friedel oscillations of the electron density in a metal around an impurity do. If the Fermi sea is quite substantial, full screening still leaves plenty of mobile carriers for tunneling conduction and the system behaves as a S–I–N tunnel junction. In this case, the differential conductance due to quasiparticles displays a maximum at a voltage $V = \Delta_g/e$, the superconducting gap. Because the number of carriers in the degenerate Sm changes with temperature, some dependence of the current density on the temperature would make the difference between a S–I–N junction and a $S - dSm$ contact.

On the contrary, if the Fermi energy level is slightly greater then the bottom of conduction band, *i.e.* the Fermi level is tiny, carriers are localized at the interface on a scale of k_F^{-1}, which is quite large ($\sim$0.1w), because the Fermi wavevector k_F is rather small compared to the metallic case. At difference with what happens in the S–I–N junction, the local density of states of the degenerate semiconductor at the interface, on which the tunneling current depends, becomes strongly energy dependent. This is what we call "imperfect screening": states within an energy range E_g about μ contribute to the screening and are unavailable for tunneling. That a very strong modification takes place in the density of states close to the interface is confirmed by the fact that the value of E_g, which is experimentally found, is up to ten times the Fermi energy in the bulk of the degenerate semiconductor. This proves that a strong modification of the band close to the interface occurs.

As an order of magnitude, let us assume that the screening in the dSm requires the localization of some conduction carriers within the depletion region up to a distance w from the interface. Crudely speaking, energies up to $\hbar^2/2m_n^* w^2$ are unavailable for tunneling. This provides a rough estimate for E_g, that is of the same order of magnitude of the measured one.

3 Model for the charge screening at the Schottky barrier

According to the picture of the previous section, the novel feature in a degenerate semiconductor is the coexistence of a small Fermi sea with a depletion region close to the interface. At very low temperature, one can assume that there is a volume up to a distance $\mathcal{L}$ from the interface in which the electron transport is ballistic. If this is the case, the depletion region acts as a barrier potential which affects the phase shifts of the incoming electrons. If the interface is well fabricated and finite size effects on the junction area can be neglected, particles conserve the parallel momentum and the problem is effectively one-dimensional.

In this section we calculate the energy dependence of the phase shifts δ of particles impinging on a barrier in one dimension. Our toy problem can be related to a change of the local density of states of the metal because of the depletion layer close to the surface, as we will explain below.

Mobile electrons impinging from the left (L) onto a square barrier of height V_0 and width $2a$, are described by plane waves with scattering amplitudes $f_{L,R}$. The one-electron wavefunction $\psi(x)$, of wavevector k, is:

$$\begin{aligned} \psi_>(x) &\propto \mathrm{e}^{\mathrm{i}kx} + f_R \mathrm{e}^{\mathrm{i}kx} \qquad x \gg 0 \\ \psi_<(x) &\propto \mathrm{e}^{\mathrm{i}kx} + f_L \mathrm{e}^{-\mathrm{i}kx} \qquad x \ll 0. \end{aligned} \tag{2}$$

Here the transmission coefficient is $T = |1 + f_R|^2$ while the reflection coefficient is $R = |f_L|^2$ satisfying conservation of flux: $T + R = 1$. In Figure 4 we plot the transmission T *vs.* energy ϵ for various barrier strengths: $V_0 a^2 = 1, 5, 10$. Its oscillations are the result of quantum interference at the scattering potential.

By placing the square barrier symmetrically with respect to the origin we can factorize the scattering into even and odd parity waves. Hence we define the even and odd parities f^l: $f^0 = \frac{1}{2}(f_L + f_R)$ and $f^1 = \frac{1}{2}(f_R - f_L)$ and the elastic $t-$ matrix $t^l = \mathrm{i}kf^l/\pi$. The matrix t is related to the $S-$ matrix according to:

$$S^l - 1 = -\frac{2\pi\mathrm{i}}{k} t^l; \qquad t^l = -\frac{k}{\pi} \sin\delta^l \mathrm{e}^{\mathrm{i}\delta^l} \tag{3}$$

because $S^l = \mathrm{e}^{2\mathrm{i}\delta^l}$, where δ^l ($l = 0, 1$) are the phase shifts for the two parities.

By solving the Schrödinger equation for the scattering amplitudes it is easy to find the phase shifts for the scattering electrons:

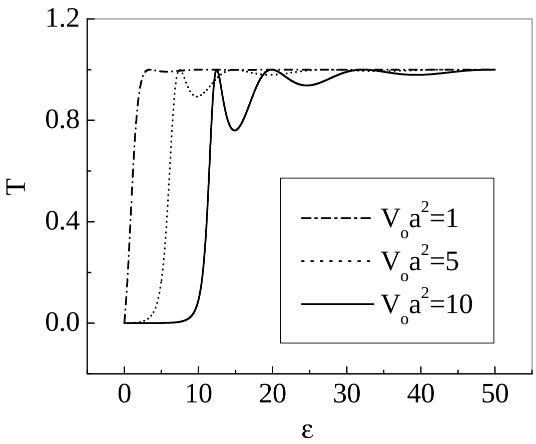

Fig. 4. The transmission coefficient T *vs.* energy ϵ in a square barrier one-dimensional model for various strengths: $V_0 a^2 = 1, 5, 10$ in $\hbar^2/2m$ units.

$$\delta^{0,1}(k) = -\frac{1}{2}\left(2ka + \arctan\left(\frac{s_-}{2}\tanh 2\kappa a\right) \pm \arctan\left(\frac{s_+}{2}\sinh 2\kappa a\right)\right). \tag{4}$$

Here is $\kappa = \sqrt{V_0 - k^2}$, with $k^2 < V_0$ ($\hbar^2/2m = 1$). The $+(-)$ sign is for the even (odd) parity and $s_\pm = \frac{\kappa}{k} \pm \frac{k}{\kappa}$.

In a metal, the scattering by a localized perturbation produces an energy change of the phase-shifts of the electron wave functions close to the Fermi surface. This can be related to the change in the density of states at energy ϵ with respect to the one of the pure metal:

$$\Delta\nu(\epsilon) = -\frac{1}{\pi}\Im m \mathbf{Tr}(\mathbf{G} - \mathbf{G_o})(\epsilon). \tag{5}$$

Here $\mathbf{G}(\epsilon) \equiv G(x, x', \epsilon)$ is the single particle retarded Green's function in the metal in presence of the perturbing potential $V(x)$, as compared to $\mathbf{G_o}$, which is the unperturbed Green's function. They are related by the scattering $\mathbf{t}$-matrix $\mathbf{t}(\epsilon) = V(1 - \mathbf{G_o}V)^{-1}$ because $\mathbf{G} = \mathbf{G_o} + \mathbf{G_o t G_o}$. Being the perturbation potential independent of the energy, it is easy to see that:

$$\Delta\nu(\epsilon) = \frac{1}{\pi}\Im m\left\{\frac{\partial}{\partial\epsilon}\ln\ \det\mathbf{t}(\epsilon)\right\} = \frac{1}{\pi}\sum_{l=0,1}\frac{\partial\delta^l(\epsilon)}{\partial\epsilon}\cdot \tag{6}$$

In Figure 5 the change in the density of states according to equation (6) is plotted *vs.* energy ϵ for various barrier strengths ($V_0 a^2 = 1, 5, 10$) in units of the bulk density of states $\nu_b = \mathcal{L}/\pi v_F$ (v_F is the Fermi velocity). The larger the barrier strength is, the larger is the depletion of one particle states close to the band bottom. States at lower energies are depleted and moved to higher energies, where they are bunched together. This is the origin of the peak in the modified density of states. The higher is the barrier, the larger is the energy range where states are depleted and moved to energies just above the dip at energy E_g.

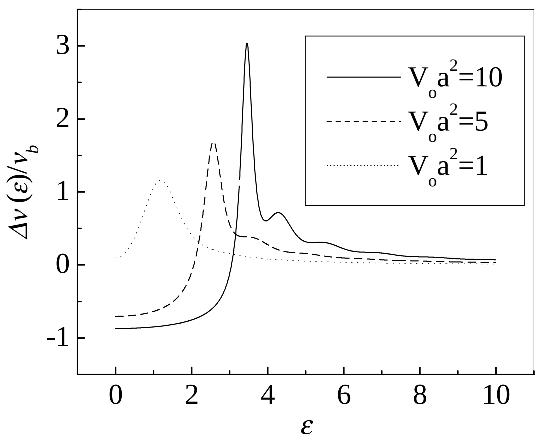

Fig. 5. The change in the density of states $\Delta\nu(\epsilon)$ induced by the barrier as derived from equation (6), for the same barrier strengths as in Figure 4, in units of the bulk density $\nu_b = \mathcal{L}/\pi v_F$.

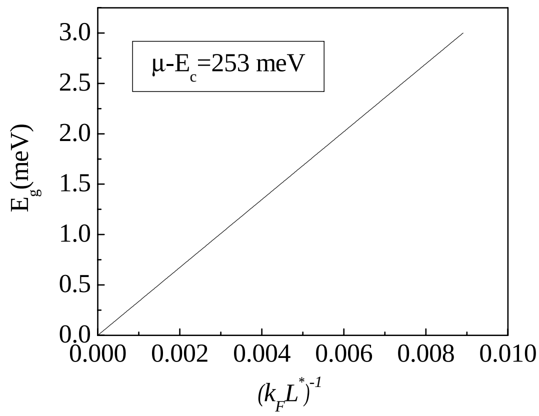

Fig. 6. Estimated dependence of the energy shift of the $R(V)$ minimum, E_g on the parameter $(k_F\mathcal{L}^*)^{-1}$ which characterizes the degenerate semiconductor (see text), as calculated from equation (8), using $\mu - E_c \simeq 253$ meV.

Full depletion is reached for $V_0 \to \infty$ when $\Delta\nu(\epsilon) = -\nu_b$. This is equivalent to say that a real gap has developed in this limit, up to energy E_g.

As a side remark, we note that $\hbar\Delta\nu(\epsilon)$ measures the so called "phase delay time", τ_{del}, of a wave-like particle that is scattered by the barrier [14]. It vanishes when the barrier is absent ($V_0 = 0$) and becomes infinite at the bottom of the band.

If the conduction band edge is not too close to the Fermi level the degenerate semiconductor is an ideal metal which conserves the particle-hole symmetry of the single particle model we are referring to. Hence the same picture applies to hole transport from L to R, immediately below ϵ_F. This implies symmetry in reversing the bias, except for the fact that the Schottky barrier is asymmetric between the two cases.

4 The shift in the minimum of R(V)

In the previous Section we have shown that, if the semiconductor is degenerate, formation of the Schottky barrier is not alternative to conduction electron screening, but both features coexist. According to the results of our one-dimensional model, if the Schottky barrier is large, an effective energy gap develops on the Sm side, at least for what tunneling is concerned: single electron states are present at those energies, but they are involved in the screening as resonances at the depletion layer charge. Apart from the possible asymmetry, this "gap" E_g inhibits the tunneling at $T = 0$ up to voltages $(E_g+\Delta_g)/e$. The Sm density of states to be put in the formula for the tunneling current should be considered as energy dependent. A maximum in the differential conductance has to be expected at $V > (E_g + \Delta_g)/e$.

Because of its large doping the semiconductor of the experiment in [3] is indeed degenerate. Therefore, one can use the Friedel sum rule to relate the change in the density of states with the charge accumulated at the Schottky barrier:

$$\int_{E_c}^{\mu_L(V)} \Delta\nu(\epsilon)\mathrm{d}\epsilon = n_{imp} A k_F^{-1} \quad (7)$$

where n_{imp} is the density of dopants which are fully ionized at the interface of cross sectional area A, within the screening length $k_F^{-1} \sim 0.1w$. If we assume, for simplicity, that tunneling states on the Sm side are fully depleted in an energy range E_g across μ, we can derive E_g from the equation:

$$\int_{\mu-E_g/2}^{\mu+E_g/2} \nu_b(\epsilon)\mathrm{d}\epsilon = 2\pi\ \mathcal{L}^* \left(\frac{2m_n^*}{h^2}\right)^{3/2} \times \int_{\mu-E_g/2}^{\mu+E_g/2} (\epsilon - E_c)^{1/2}\mathrm{d}\epsilon = n_{imp} k_F^{-1}. \quad (8)$$

Here $\mathcal{L}^*$ is the distance from the interface over which electrons keep phase coherence, so that the scattering picture of the previous section can be applied. This is the minimum between the inelastic scattering length l_{in} and the thermal length $L_T = (\hbar D/2\pi k_B T)^{1/2}$ (where D is the diffusion coefficient in the Sm).

In Figure 6 we plot E_g *vs.* $(k_F\mathcal{L}^*)^{-1}$ calculated for the experimental value $n = 10^{+19}$ cm^{-3}. We find that, for values of the ratio $(k_F\mathcal{L}^*)^{-1}$ ranging between $\approx 0.004 - 0.008$ the value of E_g obtained is of the same order of magnitude of the one measured, *i.e.* $E_{ge} \simeq 2.1$ meV.

In conclusion, we focused on the shift of the voltage position of the minimum in the $R(V)$ curve of the Nb $-$ n^{++}GaAs contact that has been observed experimentally.

We attribute this shift to effects in the charge screening at Schottky barrier of the $S - dSm$ interface, on the Sm side.

We solved a simple one-dimensional barrier model and we calculated the modification of the density of states due to the single particle scattering. Our results are interpreted as to mimic what happens on the Sm side. Close to the interface, states are unavailable for tunneling, in an energy range E_g around the Fermi energy of the degenerate semiconductor. Hence the minimum of the $R(V)$ curve is shifted at the voltage $V \sim (E_g + \Delta_g)/e$.

The value of E_g that we calculate is of the same order of magnitude of the one that is measured in the Nb $-$ n^{++}GaAs contact.

Useful discussions with A. Barone and G. Schön are gratefully acknowledged. This work was supported by Russian Foundation for Fundamental Research, Grant N 98-02-17373.

Appendix A: The Schottky barrier in the S-Sm junction

It is well known that a Schottky barrier builds up at the interface of a normal metal-semiconductor ($N - Sm$) contact.

In a macroscopic picture of thermal equilibrium, the electro-chemical potential is overall constant across the junction. This leads to band bending close to the interface on the Sm side, as shown in Figure 3, and to a depletion of carriers, with the formation of a double layer of charges of opposite sign. This, in turn, implies the presence of an electric field across the junction. When the metal becomes superconducting, this picture is not much altered on very general grounds which indeed apply to the case of Nb $-$ n^{++}GaAs junctions under study here.

In this appendix we use naive two-fluid arguments to show that the electrodynamical equilibrium of the junction is largely independent of the superconducting properties and that the depletion length w is indeed determined by normal properties. Our macroscopic arguments hold only if all quantities vary slowly in space at the interface and do not involve extra features of the superconducting coherence, such as Andreev conduction across the junction. The latter only occurs in very transparent contacts.

To discuss how the charge distribution in the metal side adjusts to screen the electric field due to a voltage gradient $-\nabla V$, we consider a normal fluid and a superfluid and we characterize their quantities by means of the label n and s respectively. The electro-chemical potential ϕ is related to the chemical potential μ simply by:

$$-e\phi_{n,s} = -eV + \mu_{n,s}.$$

In the absence of a normal current across the junction, if σ is the conductivity of the normal fluid, we have: $\vec{j}_n = -\sigma\vec{\nabla}\phi_n = 0$, what implies that $\vec{\nabla}\mu_n = -e\vec{E}$.

On the other hand, the Josephson equation for the superfluid connects the electro-chemical potential ϕ_s to the phase of the order parameter χ: $2e\phi_s = \frac{\hbar}{2e}\dot{\chi} = \text{const}$. This phase can be "gauged away" by lumping it into the vector potential of the superconducting side, so that the superfluid velocity, defined by the London equation, becomes: $\vec{v}_s = -\frac{e}{mc}(\vec{A} + \vec{\nabla}\Lambda)$ with $\Lambda = \frac{\hbar c}{2e}\chi$. Then, the stationarity condition $\dot{v}_s = 0$ straightforwardly implies $\vec{\nabla}\mu_s = -e\vec{E}$, as well.

A charge imbalance in a superconductor can be related to a change of chemical potential $\delta\mu_{n,s}$ [15]:

$$Q = -e(\chi_s \delta\mu_s + \chi_n \delta\mu_n). \tag{9}$$

Equation (9) defines the charge susceptibilities $\chi_{n,s}$ of the two fluid system. At $T = 0$ is $\chi_n = 0$, while at $T \sim T_c$ is $\chi_s = \pi\nu(\epsilon_F)\Delta_g/2k_BT$ (where $\nu(\epsilon_F)$ is the density of states at the Fermi energy). Given the sum rule $\chi_n + \chi_s = 2\nu(\epsilon_F)$ at all temperatures, we get:

$$\nabla Q = e^2(\chi_s + \chi_n)E = -2e^2\nu(\epsilon_F)\nabla V. \tag{10}$$

Finally, Poisson's equation $\nabla^2 V = -4\pi Q/\epsilon_r$ yields:

$$\nabla^2 Q - \frac{1}{\lambda_D^2}Q = 0, \qquad \frac{1}{\lambda_D^2} = \frac{4\pi e^2}{\epsilon_r}2\nu(\epsilon_F)$$

where λ_D is the Debye screening length for charges in the metal [13]. The argument also applies to the screening of the charge imbalance when the Schottky barrier forms, what proves that, at the mean field level, the presence of superconducting order in the metal is immaterial in our case.

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